

# Harmonic Analysis of Hourly Observations of Air Temperature and Pressure at British Observatories. Part I. Temperature

R. Strachey

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XII. Harmonic Analysis of Hourly Observations of Air Temperature and Pressure at British Observatories.—Part I. Temperature.

By Lieut.-General R. Strachey, F.R.S., Chairman of the Meteorological Council.

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[Plates 19-23.]

The Meteorological Council have lately published a volume entitled 'Harmonic Analysis of Hourly Observations of Air Temperature and Pressure at British Obser-It was thought preferable that this publication should be limited to the series of Tables giving the computed values of the harmonic constants, with a brief introduction explaining how the calculations had been carried out, and that the discussion of the results should be embodied in a separate memoir, which I hoped to communicate to the Royal Society, an intention which I now realize.

I have annexed to the present communication a selection of such of the Tables given in the volume referred to as appear necessary for my present purpose, and I have added a series of graphical representations of some of the results of the computations, which will facilitate the study of the subject.

I have also thought it convenient to reproduce, with some few modifications, the explanations, contained in the introduction of the same volume, of the method according to which the computations it contains were made.

The series of Tables to which I have referred give the results of computations commenced by myself, and completed in the Meteorological Office under the supervision of Mr. R. Curtis.

The computations, as originally undertaken, were designed to supply the harmonic analysis of the hourly observations of temperature and pressure made at Greenwich Observatory, in each case for twenty years, which were published in 1878.

Subsequently it was determined to extend the investigation, as a part of the regular routine of the Meteorological Office, so as to obtain the harmonic constants from the photographic records of temperature and pressure of the self-recording instruments at the seven observatories maintained by the Meteorological Office, viz., Valencia, Armagh, Falmouth, Glasgow, Stonyhurst, Aberdeen, and Kew, for a series of twelve

With a view to reducing the necessary labour, the computations of the harmonic MDCCCXCIII. --- A. 18,10,93

components, extending to the fourth order, have been made from the Greenwich data by the method proposed by myself (and explained in the 'Proceedings of the Royal Society, vol. 42, pp. 61-79), with the help of the Tables to be found in the Appendix to the "Hourly Readings of the Self-recording Instruments at the Observatories of the Meteorological Council" for 1884.

The values of the constants for the seven observatories of the Meteorological Office, extending to the third order, were obtained from the photographic records by means of the mechanical analyser designed by Sir W. Thomson, a description of which will be found in the 'Proceedings of the Royal Society,' vol. 27, p. 371, and as to the use of which and the degree of dependence to be placed on its indications reference may be made to the 'Annual Reports of the Meteorological Office,' for the years 1881, 1883, 1885, and 1890, and to the 'Proceedings of the Royal Society,' vol. 40, p. 382.

In these Tables, following the usual formula expressing the hourly value  $A_n$  in terms of its harmonic components,

$$A_n = p_0 + p_1 \cos n \cdot 15^\circ + q_1 \sin n \cdot 15^\circ + p_2 \cos 2n \cdot 15^\circ$$
, &c.,

 $p_0$  represents the mean value for the whole twenty-four hours; the coefficients of the cosines are designated by the letters  $p_1$ ,  $p_2$ ,  $p_3$ , &c., for the several orders of components; and those of the sines by the letter  $q_1$ ,  $q_2$ ,  $q_3$ , &c. The values are expressed in degrees Fahrenheit for temperature, and in inches for pressure.

The total amplitudes of the several components in the corresponding formula, involving sines only, are designated by the letter  $P_1$ ,  $P_2$ , &c., where  $P_1 = \sqrt{(p_1^2 + q_1^2)}$ , and

$$A_n = p_0 + P_1 \sin(n.15^\circ + T_1) + P_2 \sin(2n.15^\circ + T_2) + P_3 \sin(3n.15^\circ + T_3)$$
, &c.

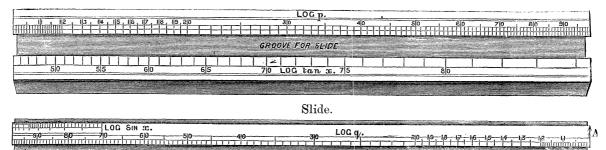
The Tables, however, instead of giving the values of the subsidiary angles T, show the epochs of the first maximum reckoned from midnight in angular measure, the letters  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ , &c., indicating the epochs for the several components. This method of stating the phase or epoch of the components has the advantage of directly indicating their relation to the hour of the day, and, therefore, to the position of the Sun, to which the diurnal variations of temperature and pressure should obviously be referred.

If it be understood that  $\tan x = p/q$ , disregarding the signs of the coefficients, then the quadrants in which T will fall when regard is had to the signs of p and q, and the relation of  $\mu$  to T and x, for the several components, are shown in the following Table.

	p+, $q+$ .	p-, q+.	p-, q-,	p+, $q-$ .
Value of ${ m T}$ $\Big\{$	0 to 90°	360° — x 270° to 360°	$180^{\circ} + x$ $180^{\circ}$ to $270^{\circ}$	$180^{\circ} - x$ $90^{\circ} \text{ to } 180^{\circ}$
Value of $\mu_1$ $\Big\{$	$90^{\circ} - T_1  90^{\circ} - x$	$450^{\circ} - T_1$ $90^{\circ} + x$	$450^{\circ} - \mathrm{T}_{1} \ 270^{\circ} - x$	$450^{\circ} - T_1 \\ 270^{\circ} + x$
Value of $\mu_2$ $\Big\{$	$45^{\circ} - \frac{1}{2}T_{2}$ $45^{\circ} - \frac{1}{2}x^{2}$	$\begin{array}{c} 225^{\circ} - \frac{1}{2} T_{2} \\ 45^{\circ} + \frac{1}{2} x \end{array}$	$\begin{array}{c} 225^{\circ} - \frac{1}{2} T_{2} \\ 135^{\circ} - \frac{1}{2} x \end{array}$	$\begin{array}{c} 225^{\circ} - \frac{1}{2} T_{2} \\ 135^{\circ} + \frac{1}{2} x \end{array}$
Value of $\mu_3$ $\Big\{$	$\begin{array}{c} 30^{\circ} - \frac{1}{3} T_{3} \\ 30^{\circ} - \frac{1}{3} x \end{array}$	$\frac{150^{\circ} - \frac{1}{3} T_3}{30^{\circ} + \frac{1}{3} x}$	$ \begin{array}{c c} 150^{\circ} - \frac{1}{3} T_{3} \\ 90^{\circ} - \frac{1}{3} x \end{array} $	$   \begin{array}{c}     150^{\circ} - \frac{1}{3} T_{3} \\     90^{\circ} + \frac{1}{3} x   \end{array} $
Value of $\mu_4$ $\Big\{$	$\begin{array}{c} 22\frac{1}{2}^{\circ} - \frac{1}{4}T_4 \\ 22\frac{1}{2}^{\circ} - \frac{1}{4}x \end{array}$	$112\frac{1}{2}^{\circ} - \frac{1}{4}T_{4}$ $22\frac{1}{2}^{\circ} + \frac{1}{4}x$	$ \begin{array}{c c} 112\frac{1}{2}^{\circ} - \frac{1}{4}T_4 \\ 67\frac{1}{2}^{\circ} - \frac{1}{4}x \end{array} $	$ \begin{array}{c c} 112\frac{1}{2}^{\circ} - \frac{1}{4}T_{4} \\ 67\frac{7}{2}^{\circ} + \frac{1}{4}x \end{array} $

The values of P and  $\mu$  were throughout obtained from the calculated values of p, q, by means of a slide rule arranged for the solution of the equations,  $\tan x = p/q$ , and  $P = p/\sin x$ , as shown below :—

# Fixed Rule.



The upper scale of the fixed rule is graduated to show logarithms of numbers, and serves to indicate the value of  $\log p$ . The graduations of the lower scale of the same rule represent log tangents to radius equal 1 on the upper scale and indicate  $\log \tan x$ .

The graduations of the lower edge of the slide rule are logarithms of numbers in the reverse order to those of the fixed rule, and indicate the values of  $\log q$ . upper edge of the slide rule is graduated to show log sines to the same radius as before, and indicates the value of  $\log \sin x$ .

The rule is used as follows:—The slide is moved in the groove till the zero point A, on its top edge, comes opposite to the graduation corresponding to  $\log p$  on the fixed rule. Then opposite the graduation corresponding to  $\log q$  on the lower edge of the slide will be found, on the scale of log tangents, the value of the angle x, or  $\log \tan x = \log p - \log q.$ 

Then, without moving the slide, the value of P will be found on the scale, showing  $\log p$ , opposite the graduation on the upper edge of the slide corresponding to  $\log p$  $\sin x$ ; or  $\log P = \log p - \log \sin x$ .

The first series of Tables in the volume spoken of deal with Greenwich air temperature, the next refer to Greenwich pressure. Then follow the results obtained by the mechanical analyser for the seven observatories, first those dealing with temperature, and next those for pressure.

The Tables for each series (a few of which only are reproduced with this paper) are arranged as follows:—

No. I. gives the values of the p, q coefficients, in the case of Greenwich, for the first four orders for each year and for each month, and for the seven observatories of the Meteorological Office for the first three orders. The mean values, in the case of Greenwich, for each series of five years, as well as the means for the whole 20 years, are also given; and for the seven observatories for the period of 12 years.

No. II. is deduced from No. I., and gives in like form the values of P and  $\mu$  for the four, or three components (as the case may be) for each year and month, and the corresponding means.

It will be observed that, in a few cases,  $\mu$  has a negative sign, which implies, as will be further explained, that the epoch of maximum is thrown back before midnight.

No. III. is derived from No. I., and gives the mean annual values of the p, q coefficients for each year, with the five-year and 20-year means as before.

No. IV. is also derived from No. I., and gives the means of the several monthly values of the p and q coefficients for the 20 years. Table A, annexed to the present paper (p. 639), reproduces the results for the Greenwich observations, and Table C those for the seven observatories.

No. V. is derived from No. II., and gives the mean annual values of P and  $\mu$  for each year, with the five-year and 20-year means.

No. VI. is also derived from No. II., and gives the means of the several monthly values of P and  $\mu$  for the 20 years.

No. VII. brings together the monthly mean values of P and  $\mu$  for the first four orders of components, for the four five-year periods over which the observations extend; the phase of maximum being in this Table stated in apparent time, instead of in mean time, as is done in the others. Table B, which is annexed to this paper, gives these results for the Greenwich observations, and Table D those for the seven observatories.

No. VIII. shows the mean monthly values of P and  $\mu$ , obtained from the mean hourly values for each month and the entire year for the whole series of 20 years, as given in No. IX. These values will be seen to differ from those supplied by No. VI., and would be the values of P and  $\mu$ , deduced from the mean values of p, q given in No. IV. The differences are due to No. VI. giving the means of the whole of the separate values obtained from the observations of the separate years and months, while the results in No. VIII. are obtained from the mean of the hourly values for

the whole period in the aggregate, in which the effects of some of the fluctuations are necessarily lost. For the same reason, the mean value for the year, in No. VIII., is not the arithmetical mean of the 12-monthly values.

No. IX. gives the mean hourly values for each month of the year, obtained from the Greenwich hourly observations. This Table is extracted from the volume of Greenwich meteorological observations for 20 years.

No. X. gives the corrections that should be applied to the Greenwich observations on account of the non-periodical variation between the initial and final midnights of the daily period. The nature of this correction, and how the non-periodical variation affects the computations of the harmonic constants, are explained in the papers before referred to; (Proceedings of the Royal Society, vol. 42).

No. XI. gives the mean value for each hour and for each year, corrected as above, and the mean for the whole 20 years.

The Tables VIII. to XI. could not be prepared in the case of the seven observatories of the Meteorological Office, inasmuch as the numerical mean values of the hourly readings had not been obtained when the calculations now published were carried out.

The 20-years series for Greenwich temperature extends from 1849 to 1868, and that for pressure from 1854 to 1873, so that the results will only be strictly comparable for the 15 years from 1854 to 1868. The five-year means compared with those for the whole series of years serve to indicate how far the mean results, obtained from relatively short periods, are likely to deviate from those got for long periods. These computations also show the degree of consistency of the computed values of the quantities dealt with, from year to year, and from month to month, in a considerable series of years.

It is necessary to state that there has occasionally been some difficulty and uncertainty attending the calculation of the mean values.

In the first place, in the series of values of the p, q coefficients, if the signs change periodically, as will be seen to be the case in the third and fourth orders, the arithmetical mean can give no true indication of the mean extent of the variations that have actually taken place. For instance, if the positive and negative quantities are in the aggregate nearly equal, the mean result would be nearly zero, though, in fact, the variations may have been very marked, and the positive and negative sums considerable.

Again, as the several components of the diurnal curve of orders above the first are recurrent, there is in some cases an ambiguity arising from the difficulty of determining whether a change of phase in successive months takes place by occurring later or earlier. This, no doubt, is mainly due to the monthly periods, for which the means are computed, being too great to admit of following the several steps of a rapid change, such as takes place about the equinoxes in the components of the third and fourth orders. A somewhat similar difficulty arises at times in the case of what may

be termed casual irregularities in the position of the phase of maximum, in dealing with some of which it becomes uncertain whether the maximum has been thrown forward or backward.

In dealing with the series of Greenwich temperature values, a doubt of this sort arises in relation to the abrupt changes in the value of  $\mu_4$  between January and February, and again between October and November, which might be attributed either to retrogression of the phase of maximum, or to progression. The uncertainty, however, in such a case hardly affects the mean monthly values, which are fairly consistent; but a doubt is introduced in determining an annual mean, the numerical value of which will depend on the conclusion adopted as to the way in which the This hesitation is unavoidable in the absence of a series of values changes take place. calculated for short periods while the change is going on.

Much greater uncertainty prevails in dealing with the barometric values than with the thermometric, the irregularities in the former being far more numerous and perplexing.

It may be noticed with reference to the quantities p, q, that their absolute amounts represent the extent of the amplitudes of the components, while their signs only indicate the phase, or epoch of maximum, in relation to the successive quadrants in each recurring series of components. A succession of signs in the order +p, +q; -p, +q; -p, -q; +p, -q, corresponds to an epoch of maximum gradually becoming later, and vice versâ. The mutual destruction of a series of positive and negative values of p, q will, therefore, merely signify that there is no true time of maximum, or that all positions are equally probable or uncertain.

When, as is not seldom the case, there are considerable variations in the values of  $\mu$ , an assumption may become necessary, in computing the mean of these values, as to the direction in which the departure of the separate values from the mean should be reckoned, whether onward or backward.

For instance, in the Greenwich tables for temperature, the value of  $\mu_2$  in the month of May for the year 1885 would by the computation appear as 85° 1'. Inasmuch, however, as will be seen from the separate values, the mean value is evidently much nearer to the commencement of the first quadrant, to which  $\mu_2$  appertains, than to its termination, it has been assumed that the value instead of being 85° 1' should be represented by  $-4^{\circ}$  59'; that is to say, that the departure of the maximum from the mean has taken place by occurring earlier and not later. All the entries of  $\mu$  in the Tables marked with the minus sign indicate that an assumption of this description has been made.

# PART I.—TEMPERATURE.

# Greenwich Temperature.

The series of Tables dealing with Greenwich temperature shows that, with considerable variations in the absolute values of all the p, q coefficients for the several months, and from year to year, as well as of the total amplitudes P, and the epochs of maximum  $\mu$ , there is still a strongly marked general consistency in the characteristics of the various elements. Reference to Table B annexed to this paper (which is Table VII. of the Greenwich series above described) shows very clearly that, with relatively few exceptions, chiefly to be found in the winter months, and among those components the absolute values of which are very small, the mean monthly epochs of maximum in the several five-year periods differ among one another by very small amounts, corresponding to intervals of time seldom as large as half-an-hour. however, the weight of the conclusions that may be drawn from these computations depends greatly on the internal evidence supplied by their greater or less consistency among themselves, it is important to dwell somewhat more on this point.

The requisite test will best be supplied by the values of  $\mu$  which indicate the times of maximum phase, and are, therefore, immediately dependent on the Sun's action, and should follow his position; and this being subject to regular periodical change in the course of the year, it would follow that if the values of  $\mu$  show corresponding regular, or nearly regular, periodical variations, they may be regarded as affording valid internal evidence of being trustworthy. The values of P, the total amplitude or variation of temperature, which are certainly subject to influences of a much more local and irregular character, will be liable to much greater and more irregular changes.

A convenient and succinct summary of the facts bearing on this point will be found in the following Table, which shows for the series of 20 years the number of times (reckoned as a percentage) in which the computed values of  $\mu$  for single months fall in the several quadrants, the hours corresponding to which are as follows:—

Time of Maximum according to Value of  $\mu$  falling in successive Quadrants.

Components.	1st quadrant.	2nd quadrant.	3rd quadrant.	4th quadrant.		
First Second Third Fourth	Midnight to 6 A.M.  ", ", 3 ", ", 2 ", ", 1.30 ",	6 A.M. to noon 3 ,, 6 A.M. 2 ,, 4 ,, 1.30 ,, 3 ,,	Noon to 6 P.M. 6 A.M. to 9 A.M. 4 ,, 6 ,, 3 ,, 4.30 ,,	6 P.M. to midnight 9 A.M. to no n 6 , 8 A.M. 4.30 ,, 6 .,		

Table showing frequency of the Value of  $\mu$  falling in the several Quadrants in different Months of the Year.

	1st component.	1	nd onent.	3	rd con	nponei	nt.		4th o	compo	nent.		
Month.	Quadrant.	Quad	lrant.		Quad	lrant.		Quadrant.					
	3	4	1	4	1	2	3	4.	1	2	3	4	
January	100 100 100 100 100 100	5 35 45	100 100 100 95 65 55 90	5	20 100 100 100 100	 75	100	15 25  5	60 5  45 100 35	25  20 50	70 95 70	.10	
July	100 100 100 100 100		100 100 100 100 100	••	100 100 84	16 40	60 100 100	25 15	40 75	55 5 10 25 10	45 95 85 10	5	

Note.—Where the 4th quadrant is shown as preceding the 1st quadrant, it is assumed that the phase has fallen back, or occurred earlier.

These figures show that in their main features the variations in the value of  $\mu$  for the second, third, and fourth components have a truly periodical character, and that as the year passes from winter to summer the maximum phase of the second and third travels backwards, that is, gradually occurs earlier in the day, while it returns in the opposite direction, becoming gradually later, in the change from summer to winter. In the fourth component corresponding changes take place between the equinoxes and the solstices.

A reference to Table B from the Greenwich series will show in more detail how well the consistency of the results is maintained in all the components.

The first component  $\mu_1$  is always in the first quadrant, i.e., between noon and 6 P.M. The variation of the five-years mean from the 20-years mean is in no month more than  $2\frac{10}{2}$ , or 10 minutes of time, and the average for all the months is less than half that amount.

In the second component the retrogression of the phase of maximum in the summer months leads to  $\mu_2$  (which from August onwards to March is always in the first quadrant, or between midnight and 3 A.M.) falling back into the fourth quadrant, that is occurring before midnight, and the greater frequency of this in May and June is therefore quite consistent with the regular periodical change which brings the mean value of  $\mu_2$  in June very nearly to zero, or midnight. The variation of the fiveyears mean from the 20-years mean is in no month greater than 6° or 24 minutes of time, and for the whole year the average variation is only 2°.3, or 9 minutes of time.

In the third component, from April to August,  $\mu_3$  is always in the first quadrant, or between midnight and 2 A.M. In September and October it changes rapidly to the third quadrant, in the last of these months 40 per cent. of the values being found in the second quadrant, and in November the value is established in the third quadrant, that is between 4 A.M. and 6 A.M., and there it remains till the end of February. In March  $\mu_3$  falls back to the first quadrant, 75 per cent. of the values being in the second quadrant and the rest in the first quadrant, with the exception of a single case which falls back to the fourth quadrant, or before midnight. The variation in any month of the five-years mean from the mean of 20 years in no case exceeds  $5^{\circ}$ , or 20 minutes of time, and the mean variation for all months is  $2^{\circ}$ ·1, or  $8\frac{1}{2}$  minutes.

The fourth component shows double maxima and minima values of  $\mu_4$ , the former occurring at the two equinoxes, the latter at the two solstices. In the winter months, November to January, the greatest frequency is in the first quadrant, between midnight and 1.30 A.M., with a considerable number before midnight or in the fourth quadrant, and a few in the second quadrant, or after 1.30 A.M. Again in May, June, and July the values of  $\mu_4$  are chiefly in the first and second quadrants, with a few cases in the fourth quadrant, while the values in June invariably fall in the first quadrant. From February to April, and again from August to October, the greatest frequency of  $\mu_4$  is in the third quadrant, or between 3 A.M. and 4.30 A.M., with deviations in the one direction to the second quadrant, and in the other to the fourth quadrant. In March and September 95 per cent, of the values fall in the third quadrant; February and April on the one side, and August and October on the other, being months of transition, between the equinoctial maxima and solstitial The largest variation of the five-years mean from that for 20 years, for any month, is 10°, or 40 minutes of time, and the average for all months 4°.3, or 17 minutes. The absolute values of the  $p_4$ ,  $q_4$  coefficients being very small, the maximum being less than  $\frac{1}{4}$ , and the average being less than  $\frac{1}{10}$ th degree Fahrenheit, the computed values of  $\mu_4$  are necessarily far more liable to error than those for the other components, and it is rather a matter for surprise that the variations should be so small, than that they should reach the actual quantities mentioned.

The values of  $\mu$  in all cases depending on that of the angle whose tangent is p/q, the consistency of the value of  $\mu$  implies consistency in the ratio of p to q. At the same time as  $P = \sqrt{(p^2 + q^2)}$  a considerable variation of the absolute value of P is quite compatible with consistent or invariable values of  $\mu$ , and such variation in P will not tend to discredit the evidence in favour of such values of  $\mu$  being trustworthy.

It may here also be noticed that as the value of P may obviously be expressed as  $p/\sin x$ , where  $\tan x = p/q$ , it follows that if the value of x, and consequently the epoch of maximum phase be regarded as fixed, for any time or place, then P will simply vary as p.

On the whole it may be regarded as sufficiently established that the computed MDCCCXCIII.—A. 4 L

components P and  $\mu$  represent well marked truly periodical variations, even, as far as the fourth order, in which the maximum value of P hardly exceeds one-third of a degree Fahrenheit.

The component of the first order, which in the winter months is more than double the magnitude of any other, and in the summer more than ten times as large as any of them, gives the dominant character to the diurnal curve of temperature. There is an obvious tendency for high or low values of P<sub>1</sub> to be associated with high and low values, respectively, in the other components, though there are many deviations from such a rule.

In the series of 20 years instances may be found in almost every month, of variations in the value of P<sub>1</sub> for different years, of as much as 100 per cent., and the maximum values of the amplitude P in all components is frequently double the Nevertheless, these irregularities for the most part disappear even in the mean of a series of five years, and the monthly mean values for 20 years are as a whole remarkably consistent, the mean difference between the five-year and the 20-year values being less than 5 per cent.

The progression of the magnitude of P<sub>1</sub> in the course of the year follows approximately the Sun's meridional zenith distance, and the empirical formula  $P_1 = 10 \cos z - 91$  gives a close approximation to the mean monthly values shown in the Tables, if a "lagging" of about eight or ten days is allowed in reckoning the zenith distance, as the following comparison will show:—

M a	Meridional	G.	Value	of P <sub>1</sub> .	Error of
Month.	zenith distance.	Cos z.	From formula.	From tables.	formula.
January February March April May June July August September October November December	73 37 66 47 55 42 44 31 36 4 29 23 29 11 34 29 45 39 57 20 67 47 73 38	·282 ·394 ·564 ·713 ·808 ·871 ·873 ·814 ·699 ·540 ·378 ·282	1·91 3·03 4·73 6·22 7·17 7·80 7·82 7·23 6·08 4·49 2·87 1·91	1·90 3·16 4·67 6·50 7·30 7·71 7·75 7·17 6·46 4·16 2·80 1·65	$+ \cdot 01$ $- \cdot 13$ $+ \cdot 06$ $- \cdot 28$ $- \cdot 13$ $+ \cdot 09$ $+ \cdot 07$ $+ \cdot 06$ $- \cdot 38$ $+ \cdot 33$ $+ \cdot 07$ $+ \cdot 26$

The amplitude of the component of the second order, P<sub>2</sub>, has two clearly marked maxima, about the time of the equinoxes, with a principal maximum at midsummer. The component of the third order, P<sub>3</sub>, varies in a converse manner, having two well marked minima at the time of the equinoxes, with a principal maximum at midsummer. In the fourth order, P<sub>4</sub> appears to combine the characters of the two former, having two maxima near the equinoxes, and a principal minimum in the winter.

The annual variation of the values of P<sub>2</sub> and P<sub>3</sub> will be represented, with a fair degree of approximation, by the expressions

$$P_2 = 1.08 + .20 \cos (\lambda + 126^{\circ}) + .41 \cos (2\lambda - 2^{\circ})$$
  
 $P_3 = .42 + .16 \cos (\lambda + 260^{\circ}) + .10 \cos (2\lambda - 172^{\circ}),$ 

in which  $\lambda$  is the Sun's longitude. The term involving cos  $2\lambda$  gives rise, in the value of P<sub>2</sub>, to the two maxima at the equinoxes, and in the value of P<sub>3</sub> to the two minima at the same seasons. The comparison of the observed and computed values of the two components for the several months is given below.

M		$P_{g}$ .			$P_3$ .	
Month.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.
January February	·94 1·26 1·34 1·21 ·71 ·56 ·66 1·11 1·71 1·51 1·17 ·76	.91 1.22 1.38 1.17 .76 .51 .70 1.18 1.57 1.55 1.17 .85 Sum . Mean .	$ \begin{array}{c} - \cdot 03 \\ - \cdot 04 \\ + \cdot 04 \\ - \cdot 04 \\ + \cdot 05 \\ - \cdot 05 \\ + \cdot 04 \\ + \cdot 07 \\ - \cdot 14 \\ + \cdot 04 \\ \cdot 00 \\ + \cdot 09 \end{array} $ $ \begin{array}{c} \cdot 63 \\ \pm \cdot 05 \end{array} $	·36 ·30 ·18 ·48 ·63 ·61 ·65 ·55 ·29 ·29 ·40 ·33	·37 ·30 ·29 ·40 ·54 ·62 ·58 ·46 ·37 ·34 ·39 42	+ ·01 ·00 + ·11 - ·08 - ·09 + ·01 - ·07 - ·09 + ·08 + ·05 - ·01 + ·09 - ·69 ± ·06

It has already been remarked that the epoch of the maximum phase of the first three components appears to be earlier in the summer and later in the winter.

In the first order the mean value of  $\mu$  is 214°, corresponding to 2 h. 26 m. p.m., the variation due to season being about 12°, or 48 minutes of time, by which amount the maximum is earlier in June and July than in December.

In the second order the first maximum in June is 20°, or 1 h. 20 m. earlier than in In the third order the difference in the same direction is 63°, or 4 h. 12 m. January. of time.

In the fourth order, as before noticed, there is some doubt as to the manner in which the change of positions of the summer and winter maximum is brought about. From March, when the first maximum occurs about 60° after midnight, or at 4 A.M., there is no doubt a retrogression, as in the other components, till June, when the maximum occurs 16° after midnight, or at 1 h. 4 m. A.M.; this is followed by a progression between June and October, when the maximum having become later, is again at about 60°, or 4 A.M. But a sudden change takes place in passing from October to November, which might be attributed either to the time of maximum rapidly advancing, that is, occurring later, or to its sudden recession, or becoming earlier; and, in whichever way it is brought about, one of the recurring epochs of maximum is established, from November to January, at about 10° after midnight. There is a like sudden change between January and February in the opposite direction, which again brings the maximum to about 60° from midnight, or 4 A.M. It will be seen that in February and November the absolute amplitude of this component is very small, and probably these sudden changes are coincident with the component becoming zero. (See Plate 23.)

Remembering that the fourth component of the diurnal curve includes a series of four undulations 90° apart, the explanation of what has just been stated, is probably to be found in a rapid change of conditions, under which the position of the first or earliest of these undulations recedes, until its place is taken by what was the second; so that as the maximum of the first undulation gradually becomes earlier, and at length occurs at 0°, or midnight, the maximum of the second undulation is approaching 90°, or 6 A.M., from which position it rapidly further recedes between January and February, in the last of which months it is found at 60°, or 4 A.M. The converse of such a process would explain the sudden change between October and November. On this hypothesis the numerical value of  $\mu_4$  in the months of November, December, and January, as given in the Tables, should be increased by 90° to render them properly comparable with the values in the other months. This would change the mean value for the year from 35° to 57°.

In proportion as the diurnal curve of temperature tends to greater simplicity, the magnitude of the component of the first order exceeds that of the others, and, in the months of May, June, and July, in which the amplitude of the first component is more than ten times that of any of the others, the temperature during the day, between the hours of 8 A.M. and 8 P.M., hardly deviates from a curve of sines. The mean temperature of the day coincides almost exactly with the temperature of the last-named hours, and the excess of the diurnal maximum over the mean hardly differs from the value of P<sub>1</sub>, which, as before shown, is directly dependent on the Sun's meridional zenith distance. (Plate 21.)

The periodical variations in the amplitudes and epochs of the second, third, and fourth components, which are indications of the departures of the hourly temperatures from the simple curve of sines represented by the first component, are, without doubt, connected with the varying length of the day and other influences dependent on the time of year, among which the direction of the prevailing winds, the greater or less transparency of the atmosphere, and the amount of vapour will have places. All of these, however, will be connected in a more or less direct manner with the Sun's position and surrounding terrestrial conditions, though an expression for this connection may be difficult to discover or define.

The relation of the epoch of the first maximum in the third component to the time of sunrise—which, though directly dependent on the Sun's declination, may, of course, equally be regarded as a function of the Sun's longitude—seems to be decidedly marked; the former occurring, on the average of the whole year, at an interval of 48°, or 3 h. 12 m. earlier than the latter, or, what is the same thing, the first minimum occurring 12° or 48 minutes after sunrise, the mean deviation of the interval from that average being only 7°, or 28 minutes of time. A reference to Table B will illustrate this.

The periodical variation in the value of  $\mu_3$  leads to the third component having a positive maximum about 1 P.M. during the winter months, from October to February, which will be accompanied by maximum negative values four hours earlier and four hours later, corresponding with the reduced temperature in the mornings and afternoons of the shorter days. (Plate 23.)

In the summer months, from April to August,  $\mu_3$  has a maximum negative value about 1 P.M., instead of a positive maximum as in winter; and this, being accompanied by two positive maxima, one four hours earlier and the other four hours later, will in like manner correspond to the higher temperature in the mornings and afternoons of the longer day.

It will be seen that the positions of the midsummer and midwinter maximum phases correspond respectively to days of 16 hours with nights of 8 hours, or days of 8 hours with nights of 16 hours, and that at these seasons, when the variations of temperature due to these differences are greatest, the values of the amplitude of the component are also at a maximum. At the equinoxes, with a 12 hours day and night, the amplitude of the component has a minimum value, and the transition of the position of the maximum phase takes place as already described.

It might, perhaps, be expected that the change in the position of the maximum phase of the fourth component,  $\mu_4$ , would take place in an analogous manner, in connection with days and nights of 6 and 18 hours duration respectively, corresponding to the epoch of the component. But such days and nights will only occur in higher latitudes than those of our observatories, and no data are available by which to test such a suggestion.

Although the several harmonic components of the temperature curve cannot be regarded as indication of specific physical efficient causes, acting at definite periodical harmonic intervals, the graphical representations of the series of monthly temperature curves and their components, which are given in the plates accompanying this paper, present some points to which attention may usefully be drawn.

As before noticed, the deviation of the day temperature from the first component is extremely small in the months of May, June, and July, during which between 8 A.M. and 8 P.M. the other components nearly cancel one another. From September to March however, the temperature during the day hours rises considerably above the first component, the second and third component either being both positive and

operating in the same direction, or the third component if negative being overpowered by the other two, which from 10 A.M. to 4 P.M. are always positive.

The temperature curve between the hours of 4 A.M. or 5 A.M. and 7 A.M. or 8 A.M. always falls below the first component.

The first component having its maximum about 2 P.M. is always negative between 8 P.M. and 8 A.M.

The second component having a day maximum which varies between 11.30 A.M. to 1.30 P.M. is always negative between 4 A.M. and 8 A.M. In the summer months when the third component is largest, it is also negative between 4 A.M. and 7 A.M., and at other seasons, when it would operate in an opposite direction, its relatively small magnitude renders its effects unimportant.

The second component always having a maximum near noon will also have another near midnight.

In the summer months when the third component has a minimum shortly following noon, it will have a maximum shortly following midnight, that is nearly at the same time as the second component, and at this season the night temperature is found to be considerably above the first component. In the winter months the third component becomes negative during the hours when it was before positive, and the departure of the night temperature from the first component is much smaller.

In the summer months the time of morning mean temperature is nearly when the first component becomes zero, the second and third components then balancing one another.

In the winter the hour of morning mean temperature is later, and occurs when a positive value of the first component equals a negative value of the second.

The time of afternoon mean temperature throughout the year is somewhat either before or after 7 P.M., and almost exactly coincides with the time when the first and second components are equal but with opposite signs, the former being positive and the latter negative.

In the summer months the whole of the components are negative between the hours of 3 A.M. and 6 A.M., corresponding with the absolute minimum about this time.

In the winter the third and fourth components are not negative till after 6 A.M., while the negative branch of the first component is extended almost to 9 A.M., and the absolute minimum is found correspondingly later between 7 A.M. and 8 A.M.

The hour of absolute maximum is nearly coincident with that of the dominant first component, which is always between 2 P.M. and 3 P.M., the modifications due to the second component, which also has a maximum between noon and 2 P.M., being of secondary importance.

Sunrise in December is about an hour and-a-half before the time of mean temperature, while in June it is more than four hours before that time.

On the other hand, while sunset in December is rather more than three hours before the time of mean temperature, in June it is about half-an-hour after that time.

Also it will be found that in January whereas the temperature is above the mean of the whole 24 hours for not more than  $8\frac{3}{4}$  hours, in June it is above the mean for nearly 12 hours.

It may here be pointed out that the *rationale* of some of the empirical rules for obtaining the mean daily temperature, from a limited number of observations made at certain stated hours, is supplied by reference to the harmonic expressions for the hourly deviations of the temperature from the mean of the whole day.

In the first place it is obvious, that by adding together the harmonic expressions for any two hours, twelve hours apart, the whole of the *odd* components disappear, and that the sum is twice the mean value added to twice the sum of the *even* components of the selected pair of hours, which are respectively equal to one another. Disregarding the components above the fourth order, as may be done in practice, if the hours selected be such that the component of the second order is zero, which will be the case when the hours correspond to  $45^{\circ} + \mu_2$ , or  $135^{\circ} + \mu_2$ , then half the sum of the observed temperatures at the selected pair of hours will be equal to the true daily mean added to the value of the component of the fourth order for the selected hours, which at English stations will hardly amount to so much as half a degree Fahrenheit. At Greenwich it will be found that in January the mean of observations at 4.30 A.M., and 4.30 P.M., or at 10.30 A.M., and 10.30 P.M., differ by less than 0°·1 from the true mean, and a similar result will be obtained in June by taking the mean of observations at 3 A.M., or 9 A.M., and at the corresponding afternoon hours.

For like reasons it will be seen that by taking the mean of observations at any four hours, at equal intervals of six hours, not only will the whole of the odd components disappear, but those of the second order also, so that the result will only differ from the true mean by the amount of the component of the fourth order for the selected hours. As the fourth component disappears at hours when  $\mu_4 \pm 22\frac{1}{2}^{\circ}$  equals 0° or 180°, the hours at Greenwich which will give the best result will be found to be 2, 8, 14, and 20, or 5, 11, 17, and 23.

Again, if the mean of any three hours, at equal intervals of eight hours, be taken, it will be seen that the sums of the first, second, and fourth components disappear, and that the result will only differ from the true daily mean by the amount of the third component for the selected hours, which at English stations will hardly exceed three-quarters of a degree. The best result will be obtained when the selected hours correspond to  $\mu_3 \pm 30^{\circ} = 0^{\circ}$ , or 180°, when the third component becomes zero, and at Greenwich for the greater part of the year this condition will be nearly complied with by the hours 3, 11, and 19, or 7, 15, and 23.

It will be readily understood that the true reason for such close approximations being so easily obtained, is the great preponderance of the first and second components of the temperature variation over all the rest.

# Temperature at the Seven Observatories.

The mechanical analyser, by means of which the harmonic components of the temperature curves for these observatories have been obtained, is only adapted for computing the coefficients of the first three orders, so that the constants for the fourth order are not given in the published tables.

The values of P and  $\mu$  were calculated from the values of the p, q coefficients, which are obtained by means of the instrument, in the same way as those for Greenwich.

An inspection of the table C and D, which are annexed, and contain a summary of the results, will at once show that in their main characteristics the results closely resemble those for Greenwich, and for this reason it will not be necessary to discuss them at great length.

The amplitude of the component of the first order  $P_i$  is, however, in all cases below that of Greenwich, the lowest values being those of Valencia and Falmouth, no doubt due to their position on the coast, with means for the year of 2°.28 and 2°.35, compared with 5°.10 at Greenwich.

The Kew values most resemble those of Greenwich, but the mean maximum is more than one degree less, and the mean for the year about half a degree less. differences are probably in a considerable degree due to differences of exposure, and local conditions of soil, &c.

The mean values of  $\mu_1$  for the year referred to local apparent time, lie between 205° and 220°, that for Greenwich being 214°. The summer values are somewhat below the mean, and the winter values somewhat above it, as is the case at Greenwich.

The amplitude of the first component conforms approximately, but not so closely as in the case of Greenwich, to the sine of the sun's meridian altitude, with a flattening of the curve in the summer months, and a tendency at some stations to a maximum value in May.

The component of the second order in all cases shows the double maximum at the equinoxes and the minimum value in summer. The numerical values of  $P_2$  are, as a rule, below those at Greenwich, the yearly means varying between 0°.84 and 0°.61, against 1°.08 at Greenwich; Kew, as before, most closely resembling Greenwich.

The values of  $\mu_2$  generally follow the same law as at Greenwich, the epoch of maximum being earlier in the summer and later in the winter, this character being specially marked at Aberdeen, where the summer value of  $\mu_2$  is  $-44^{\circ}$ , and the winter value 20°, a difference of 64°, or more than four hours of time, a result no doubt connected with the higher latitude of that station, the earlier sunrise and the longer day.

The component of the third order also in all cases closely resembles that at Greenwich, P<sub>3</sub> having the double minimum at the two equinoxes, and a maximum in The mean yearly value is again somewhat below that for Greenwich, varying from 0°.23 to 0°.38, the Greenwich value being 0°.42.

The values of  $\mu_3$  accord very closely with those at Greenwich where the mean yearly value is 41°, the mean yearly values at the seven observatories ranging between 37° and 43°. The variations of the epochs of maximum from winter to summer are at all stations well marked, and at Aberdeen, as was the case with the component of the second order,  $\mu_3$  shows an exceptionally low value in the summer, differing about 70°, or nearly five hours of time, from the winter epoch.

The expressions for the annual variations of the values of  $P_2$  and  $P_3$ , derived from the means of the observations at the seven observatories, differ little from those before given for the Greenwich values, allowance being made for the smaller amplitude of all the components. They are as follows:—

$$P_2 = .68 + .14 \cos (\lambda + 105^{\circ}) + .21 \cos (2\lambda - 8^{\circ}),$$
  
 $P_3 = .30 + .13 \cos (\lambda + 260^{\circ}) + .07 \cos (2\lambda - 166^{\circ}).$ 

The comparison of the observed and computed monthly values of the components for several months is given below.

<b>N</b> F 11		$P_2$ .		$\mathrm{P}_3$ .						
Months.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.				
January February March April May June July August September October November December		62 ·78 ·85 ·63 ·49 ·33 ·40 ·65 ·90 ·94 ·78 ·63 Sum	$ \begin{array}{c} \cdot 00 \\ \cdot 00 \\ - \cdot 08 \\ - \cdot 12 \\ + \cdot 07 \\ \cdot 00 \\ + \cdot 05 \\ - \cdot 01 \\ - \cdot 07 \\ - \cdot 08 \\ - \cdot 01 \\ + \cdot 04 \end{array} $	·22 ·17 ·12 ·32 ·53 ·43 ·42 ·40 ·27 ·15 ·24 ·22	22 ·18 ·19 ·30 ·41 ·46 ·41 ·33 ·25 ·23 ·28 ·26 Sum	$ \begin{array}{c} \cdot 00 \\ + \cdot 01 \\ + \cdot 07 \\ - \cdot 02 \\ - \cdot 12 \\ + \cdot 03 \\ - \cdot 01 \\ - \cdot 07 \\ - \cdot 02 \\ + \cdot 08 \\ + \cdot 04 \\ + \cdot 04 \end{array} $				
		Mean	•04		Mean	.04				

On the whole it may be regarded as well established by the substantial consistency of these results, that they truly indicate the chief characters of the diurnal inequality of temperature in the British Isles; and they leave no room to doubt that the variations which recur with such remarkable regularity from year to year, and are observed at so many different places, extending to quantities so small as those represented by the third and fourth order of harmonic components, are due to the operations of determinate physical causes, the effects of which this system of analysis enables us to distinguish and record with remarkable and unexpected precision.

I am not in a position to indicate, further than has already been done, any direct or precise relation between the observed thermometric results thus recorded, and the varying place of the Sun, on which in combination with the latitude and local influences of different descriptions they doubtless depend. The intermittent character of solar heat introduces a discontinuity of action which renders its connection with the resulting phenomena specially difficult of representation by algebraical expressions.

Acting on a suggestion of Professor G. Darwin, I have calculated the harmonic constants that would reproduce a curve indicating an intermittent action such as that of the Sun, continuing only during a portion of the day, and commencing and ending abruptly at sunrise and sunset. The results are of some interest, and serve to throw light on the character and signification of the harmonic components of temperature that have been under discussion.

Such a calculation obviously disregards all cooling effects, and only deals with the Sun's heating action, which I have assumed to be proportional to the sine of his altitude; and with a view to obtaining figures in some degree comparable with those obtained from actual observation, and following the analogy of the value of P<sub>1</sub>, as previously pointed out, the total heating effect has been taken to be ten times the sine of the Sun's altitude, the power of a vertical Sun being that represented by ten.

On this assumption I have calculated the Sun's altitude for each hour of the day for midwinter, the equinox, and midsummer for certain selected latitudes, and corresponding heating effects. These computations supply a series of hourly values having their maximum at noon and becoming zero at sunrise and sunset, and disappearing so long as the Sun is below the horizon. Treating these in the usual manner, the resulting harmonic components as far as the fourth order are shown in the following Table F:—

TABLE F.

	Sections and process on a few management had the	Midw	inter.	il lannaudhan nàime anns an seireachtaine.		Equino	ox.			Midsuı	nmer.	TO ANTI-OLOGICA AND A STATE OF THE STATE OF	
Latitude.		Comp	onents.	alled the a mark beautiful the 1122 piles on 185 to 16		Compone	ents.	No. 60 ( ) and address consistent (	Components.				
	P <sub>1</sub> .	P <sub>2</sub> .	P <sub>1</sub> .	$\mathbf{P}_{2}$ .	P <sub>3</sub> .	P <sub>4</sub> .	P <sub>1</sub> .	$P_2$ .	P <sub>3</sub> .	P <sub>4</sub> .			
0 20 30 40 45 51½ 65	$\begin{array}{r} -4.58 \\ -3.40 \\ -2.76 \\ -1.92 \\ -1.51 \\ -1.01 \\ 0 \end{array}$	+1.92 $+1.70$ $+1.42$ $+1.03$ $+.77$ $0$	0 ·26 ·28 ·43 ·42 ·39	- ·40 - ·30 - ·11 - ·06 + ·03 + ·12	-5.06 -4.74 -4.35 -3.86 -3.52 -3.15 -2.21	+2.14 $+2.02$ $+1.68$ $+1.60$ $+1.49$ $+1.33$ $+.95$	0 0 0 0 0 0	- 45 - 43 - 37 - 35 - 35 - 30 - 20	$ \begin{array}{r r} -5.25 \\ -5.27 \\ -5.12 \\ -5.19 \end{array} $	+1.55  +1.22  +1.05	+ ·29 + ·38 + ·43 + ·37	$ \begin{array}{r}40 \\30 \\21 \\05 \\ +.06 \\ +.18 \\ +.09 \end{array} $	

As from the nature of the hypothesis adopted the diurnal curve analysed is in all cases symmetrical on either side of noon, the values of q are all zero. Also the values

of P and p become identical; the negative sign in the above table signifies that the component has a negative maximum value at midnight; and the positive sign that the component has a positive maximum value at the same hour. The values of  $\mu$  for the several components will be seen to be as follows:—For the first component  $\mu_1 = 180^{\circ}$ , or noon; for the second  $\mu_2 = 0^{\circ}$ , or midnight; for the third  $\mu_3 = 0^{\circ}$  for the summer, and  $\mu_3 = 60^{\circ}$  for the winter, the change taking place at the equinox when the component becomes zero; for the fourth, in the lower latitudes,  $\mu_3=45^{\circ}$  at all seasons, in the higher latitudes at the equinox  $\mu_4 = 45^{\circ}$ , passing to  $0^{\circ}$  both in the winter and summer.

For comparison with the above results, are given below the components of the diurnal temperature curve as actually observed at certain selected stations, varying in latitude, for the three seasons of the year.

Table G.

		Mid-w	inter.			Equi	nox.		Mid-summer. Components.				
Stations.		Compo	nents.			Compo	nents.	an Taraha kara mengerinan kanangan mengerinan kanangan mengerinan kanangan mengerinan kanangan mengerinan kanan					
	P <sub>1</sub> P <sub>2</sub> . P <sub>3</sub> . P <sub>4</sub> .				$P_1$ .	P <sub>2</sub> .	$P_3$ .	$P_4$ .	P <sub>1</sub> .	P <sub>2</sub> .	P <sub>3</sub> .	P <sub>4</sub> .	
Singapore. Lat. 1° 15′ N.	-5.02	+1.78	<b>-</b> ⋅28	18	-6.47	+2.04	+ 68	- 48	-4·62	+1.54	+ ·32	<b>-</b> ·51	
Hong Kong. Lat. 22°18′ N.	<b>-2</b> ·39	+1:01	<b>-</b> ∙67	<b>-</b> ⋅81	-1.98	+ .70	02	-:12	-1.86	+ .63	+.09	<b>-</b> ·13	
Lyons. Lat. 45°46′ N.	<b>−</b> 2·51	+1.01	<b>-</b> ·41	+.07	-6.26	+1.49	+:11	<b>-</b> ∵31	-7.55	+ .76	+.59	+ 16	
Greenwich. Lat. 51°30′ N.	-1.65	+ .76	:33	+.09	4:67	+1.34	+:18	<b>-</b> ·23	-7.71	+ .56	+.61	+ 21	
Fort Rae. Lat. 62°40′ N.	-1.09	+ .67	<b>-</b> ∙31	+ 29	-7:71	+1.90	+ 42	:50	-6.01	+ :59	+.19	+.06	

The signs indicate the sign of the components at midnight.

It will hence be seen that while the intermittent curves representing the heating power of the Sun, under varying conditions of meridional altitude and length of day, are fairly well reproduced by combining the components of the first four orders having the values shown in Table F, these components bear a very close resemblance to those shown in Table G, obtained from the observed temperatures of places at the latitudes and seasons corresponding to those adopted in the computations from the intermittent curves.

The conclusion is unavoidable, that although both in the real and hypothetical cases the harmonic components, when combined, are truly representative of the peculiar

forms of the curves from which they were derived, this affords no true indication of the existence of distinct physical influences operating in recurring cycles of 24, 12, 8, and 6, corresponding to the four orders of components; but that the results are, to a great extent, due to the particular form given to the analysis.

The component of the first order represents a variation from the mean value, which is symmetrical on either side of the epoch of its maximum, and necessarily involves a negative departure from the mean, equal to a corresponding positive departure at the hour of maximum, and twelve hours distant from the maximum.

As the diurnal curve of temperature is not symmetrical in relation to the mean value, the maximum day temperature being much more in excess of the mean than the minimum night temperature is below it, the first component, which is symmetrical in this respect, must be modified by the other components; and that of the second order, which has one of its maxima not far removed from the epoch of minimum of the first component, supplies the chief portion of the compensation necessary to correct the excess of that minimum over the true minimum.

Further, from the character of the analysis, the third component will be zero when the diurnal curve is symmetrical, but with contrary signs on either side of the hours half-way between noon and midnight, that is when the length of the day and night are each 12 hours. Any departure from this equality will introduce a component of the third order; with the result, that with a day shorter than 12 hours, one maximum phase will be between 6 A.M. and 6 P.M., and the other two in the hours before 6 A.M. and after 6 P.M.; while with a day longer than 12 hours, two maxima will be between 6 A.M. and 6 P.M., and the other in the hours after 6 P.M. and before 6 A.M. the former case the two negative phases of the component will correspond with the reduced temperature in the morning and afternoon of the shorter day, and in the latter the two positive phases will correspond with the increased heat of the mornings and afternoons of the longer days.

It will also be apparent that with a day of 8 hours, and a night of 16 hours, or vice versa, the epochs of the third component would tend to synchronize with the duration of the night and day, and its amplitude might be expected to be then greater, which is seen to be the case.

Analogous considerations will apply to the component of the fourth order. positive maximum phase at or near midnight would give two maxima, near 6 A.M. and 6 P.M., respectively, corresponding to the long days in high latitudes in the summer, and also two negative minima at or near 9 A.M. and 3 A.M. respectively, corresponding to the short days in winter, in high latitudes. The Tables F and G both show P<sub>4</sub> with a positive sign under the circumstances just stated.

Again a negative phase at or near midnight would correspond to two negative phases at or near 6 A.M. and 6 P.M., respectively, which would correspond to the days and nights of nearly 12 hours in the lower latitudes and at the equinoxes, which con-

ditions are also shown to exist by the two Tables before referred to, the sign of P<sub>4</sub> being in all cases negative at the equinox.

In connection with this view of the temperature components it may be noticed that if instead of reckoning the epochs of maxima as the phase nearest to midnight, that nearest noon were adopted, it would be found that the usual time of maximum phase of all the components is not far removed from noon, affording strong evidence that all these phases are closely dependent on the passage of the Sun over the meridian.

For Greenwich the following would be the results:—

	December.	March.	June.
1st component	222	215	210
	200	198	181
	194		(193)
	190	(193)	196

In case of the third and fourth components the figures enclosed in brackets ( ) are epochs of minimum.

In conclusion, I would observe that in the course of the preparation of the data on which this discussion has been based, I have computed the harmonic components of the temperature curves of many other places besides those here specially noticed, and that the results obtained from them entirely confirm those now set forth. I am not, however, in a position at the present time to present any satisfactory details of these investigations, involving as they do very tedious computations, which I have not yet been able to carry out in a systematic or complete shape.

The importance of the subject in connection with the study of the diurnal variations of atmospheric pressure cannot be overstated, and in the second part of this memoir, which will be devoted to that question, I shall have occasion to revert to it. while I would invite the co-operation of persons interested in this branch of physics in the collection of data relating to temperature as complete as those now available in the case of atmospheric pressure.

Note.—In 1879 Sir G. Stokes communicated to the Meteorological Council, of which he was then a member, copies of letters addressed by him to Sir G. Airy, containing the results of computations of the harmonic components of the Greenwich diurnal temperature curve, made in the ordinary manner, which differs somewhat from that adopted by me. These results supply the coefficients up to the fourth order for each of the 20 years of the Greenwich series, separately, and those of the fifth and sixth orders for the mean of the 20 years taken together. They are almost identical with the values given in the volume of tables published by the Meteorological Council, and the comparison of the means of the two sets of computations may be of some interest.

	$p_1$ .	$q_1$ .	$p_{2}$ .	$q_{2}$ .	$p_3$ .	$q_3$ .	$p_4$ .	$q_4$ .	$p_5$ .	$q_5$ .	$p_6$ .	$q_{6}.$
Sir G. STOKES Meteorological Tables	-4·280 -4·282						ĺ		<b>-</b> ·001	026	003	<b>-</b> ·008
	P <sub>1</sub> .		$P_2$ .		$P_3$ .		$\mathbf{P_4}.$		F	5.	F	6.
Sir G. STOKES Meteorological Tables	5·0 5·0		_	047 046	-	35 34		45 47	.0	26	.0	09

It has not been thought worth while to repeat the detailed figures for the separate years, as they are virtually identical with those given in the Meteorological Office Tables.

Referring to a comparison that he had made between the mean observed hourly temperatures and the values obtained by computation from the daily means and the harmonic coefficients, Sir G. Stokes remarked that "the minute fourth order is swallowed up, even in a mean of 20 years' series, by the uncertainty in the amount of the diurnal fluctuation, not to speak of the much larger uncertainty in the mean temperature for a day. Nevertheless, it appears from the figures, that though this order is so minute, and though its effect on the mean temperature at any particular hour is trifling compared with the average variation from year to year of the same hour, it still emerges roughly with tolerable certainty from the mean of a good many years."

This conclusion is fully confirmed by the more complete examination of the results obtained from the mean *monthly* values, much of the strong evidence of the periodicity represented by the minute fourth order of components being lost by dealing with the mean *yearly* values.

Observations from 1849 to 1868.

# Table A.—Mean Values (reckoned from Mean Midnight) of the p, q Coefficients for the First Four Orders of Harmonic Components of the Diurnal Inequality of Temperature at Greenwich, for each Month of the Year, obtained from 20 Years'

HOURLY OBSERVATIONS OF AIR TEMPERATURE AND PRESSURE.

Mean Months.  $q_2$ .  $q_3$ .  $p_4$ .  $q_4$ .  $p_1$ .  $q_1$ .  $p_2$ .  $p_3$ . temp. T.  $\overset{\circ}{:}_{26}$ °64 °69 04  $^{\circ}24$  $^{\circ}04$  $-1^{\circ}45$ -1.2038.9 January. -2.39-2.06.88 .89 ·23 .17  $\cdot 00$ .07 39 7 + + February + + -2.82+1.02.06 .13  $\cdot 12$  $\cdot 19$ 41.5March -3.71+++ ·87 ++++ .56 .27 .36 .15 47.2+1.04\_ + .08 April. -5.46-3.21+++ -3.61.63 .17 •50 .38 .01 + .09 52.7 -6.34May . + -6.69++  $\cdot 39$ .10 .17 59.8June-3.83+ •50 .04 •46 + .23 .38 .01  $\cdot 12$ .50 62.5-6.58-4.08.57 July ·53 ·83 ·74 -6.06-3.83.95 + .38 + .39 .16 + .01 61.9August .25 .23 .23 .06 -5.60-3.21+1.49+ 57.5September -2.04+1.35+  $\cdot 25$ .04 - .10 .10 50.9-3.61October . ++ -1.60+ .96 .67  $\cdot 35$ .18  $\cdot 03$ + .01 42.8November -2.29+ .05 .06  $\cdot 25$  $\cdot 21$ + 40.8-1.22-1.07 $\cdot 59$ + .47 December + .066 -- .012 49.69 Mean 20 years -4.282-2.738+ .885 + '557 + '118 -046

Table B.—Values of the Amplitude P, and the Apparent Time of the First Maximum  $\mu$ , for the First Four Orders of Harmonic Components of the Diurnal Inequality of Temperature at Greenwich for each Month of the Year, for Five-Yearly Periods between 1849 and 1868.

	Five- yearly periods.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
$P_1$	1st 2nd 3rd 4th	1·90 2·00 1·99 1·71	3·25 3·50 3·17 2·71	4·94 5·05 4·42 4·27	5·80 7·19 6·05 6·94	6·97 7·25 7·24 7·74	7·76 8·13 6·55 8·40	7·39 7·43 7·62 8·57	6·85 7·76 6·75 7·30	6·20 6·98 5·82 6·74	3·94 4·28 3·93 4·47	2·74 2·61 2·89 2·97	1·44 1·74 1·79 1·62	4·93 5·33 4·86 5·29
$P_2$	1st 2nd 3rd 4th	·97 1·00 1·00 ·80	1·20 1·33 1·32 1·18	1:40 1:43 1:26 1:27	1·04 1·45 1·07 1·27	·66 ·75 ·82 ·59	·64 ·63 ·49 ·47	·72 ·63 ·57 ·72	1·05 1·32 ·98 1·07	1·57 2·01 1·36 1·83	1·50 1·59 1·43 1·66	1·09 1·15 1·22 1·22	·78 ·81 ·77 ·69	1·05 1·18 1·03 1·06
$P_3$	1st 2nd 3rd 4th	·40 ·34 ·34 ·35	·28 ·31 ·28 ·32	·16 ·22 ·14 ·19	·49 ·49 ·47 ·45	·65 ·63 ·48 ·77	·73 ·58 ·43 ·69	·64 ·62 ·61 ·72	·56 ·52 ·46 ·67	·27 ·37 ·30 ·23	·23 ·35 ·23 ·35 ·35	·40 ·38 ·40 ·43	·28 ·35 ·36 ·33	·43 ·43 ·38 ·46
$P_4$	1st 2nd	·09 ·08 ·11 ·09	·12 ·11 ·08 ·11	·27 ·27 ·18 ·19 ·23	·18 ·24 ·19 ·19 ·20	·08 ·14 ·12 ·13 ·12	·15 ·24 ·17 ·29	·18 ·15 ·19 ·17	·20 ·15 ·15 ·19 ·17	·34 ·36 ·32 ·33	·14 ·15 ·16 ·23	·07 ·06 ·05 ·11	·10 ·08 ·10 ·09	·16 ·17 ·16 ·18

# Table B (continued).—Values of the Amplitude P, and the Apparent Time of the First Maximum $\mu$ , for the First Four Orders of Harmonic Components of the Diurnal Inequality of Temperature at Greenwich for each Month of the Year, for Five-Yearly Periods between 1849 and 1868.

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	Five- yearly periods.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Year.
$\mu_1$	1st 2nd 3rd 4th	222° 216 213 217	217° 216 216 220	214° 217 214 215	212° 212 214 113	211° 210 212 210	208° 211 209 211	208° 211 211 212	213° 213 212 213	211° 212 210 211	212° 212 213 216	218° 220 218 218	223° 226 217 222	214° 215 213 215
	Mean	217	217	215	213	211	210	210	213	211	213	219	222	214
$\mu_2$	1st 2nd 3rd 4th	22 19 23 20	21 17 19 20	18 17 22 16	9 14 12 17	9 6 17 – 2	- 8 6 9 - 1	0 8 11 13	16 15 17 14	19 15 13 16	19 18 18 17	22 20 20 21	18 20 23 20	14 14 17 14
	Mean	21	19	18	13	8	1	8	16	16	18	21	20	15
$\mu_3$	1st 2nd 3rd 4th	75 74 73 72	69 70 69 66	40 30 40 49	14 19 15 22	14 14 11 13	16 12 13 12	12 12 10 11	15 18 13 16	21 36 23 26	66 68 68 62	75 73 72 70	76 73 72 77	41 41 40 41
	Mean	74	69	40	18	13	13	11	16	27	66	73	74	41
$\mu_4$	1st 2nd 3rd 4th	12 14 10 5	56 57 65 68	58 57 56 61	47 55 48 61	25 27 16 24	16 20 17 11	23 30 24 7	46 47 38 46	58 60 57 57	62 62 51 62	25 16 - 8 9	12 14 0 15	58 60 53 57
	Mean	10	62	58	53	23	16	21	44	58	59	11	10	57
	Time of sunrise	118°	106°	91°	77°	66°	58°	61°	72°	87°	101°	115°	122°	

Table C.--Mean Values (reckoned from Greenwich Mean Midnight) of the p, q Coefficients for the First Three Orders of Harmonic Components, of the Diurnal Inequality of Temperature at the Seven Observatories of the Meteorological Office, MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES TRANSACTIONS SOCIETY SOCIETY

for each Month of the Year, obtained from Twelve Years' Observations from 1871 to 1882.

		1		
Кем.	0° 19′		1	60.+
Аретдееп.	2° 6′		+ + + + + +	90.+ 20.+ 60.+
Stonyhurst.	2° 28′		+ + + + + +	40.+
Glasgow.	4°18′	$p_3$ .	+ + + + + +	60.+
Falmouth.	5° 4′	Property of the Control of the Contr	+ + + + +	+.05
Armagh.	6° 39′		+ + + + +	03
Valencia.	10°18′	•	+ + + + + +	+ .04
1		 		
Kew.	0° 19′		++++++++++++++++++++++++++++++++++++++	+ .58
Aberdeen.	2° 6′	-	++++++++++ 7.6.6.6.0.1.4.7.7.6.6.0.1.4.7.7.6.6.1.4.7.6.6.1.4.7.7.6.1.4.7.6.1.4.7.6.1.4.	+.54 +.43
Stonyhurst.	<b>2</b> ° 28′		+++++++++++ 4477941233989999999999999999999999999999999999	+ :54
Glasgow.	4°18′	$p_2$ .	++++++++++ 24; i i i i i i i i i i i i i i i i i i i	+.42
Falmouth.	·5。 4		++++++++++++++++++++++++++++++++++++++	+.52
.dgsmrA	6° 39′		++++++++++ 1.5.4.6.4.4.4.5.0.8.8.6.2.5.0.8.6.7.5.0.7.5.0.8.6.7.5.0.	+.55
Valencia.	10°18′		++++++++++++ 22448 2448 25248 2609 2688 888	88: +
Kew.	0° 19′		1.27 1.1.27 1.64 1.64 1.64 1.64 1.64 1.64 1.64 1.64	-3:38
Aberdeen.	5°,		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-2.21
Stonyhurst.	2°, 28′,	CONTRACTOR DE CO	11.4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 +	, , , , , , , , , , , , , , , , , , ,
Glasgow.	4°18′	$p_1$ .		-2.47
Falmouth.	50 中	·	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	-2.03
Armagh.	6° 39'		7. 1. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4.	-2.45
Valencia.	10°18′			-1.84
	Longitude correction local mid- night later than Greenwich mid- night	Months.	January February March April May June July September October November December	Mean 12 years .

PHILOSOPHICAL TRANSACTIONS

Table C (continued).—Mean Values (reckoned from Greenwich Mean Midnight) of the p, q Coefficients for the First Three Orders of Harmonic Components, of the Diurnal Inequality of Temperature at the Seven Observatories of the Meteorolo-

gical Office, for each Month of the Year, obtained from Twelve Years' Observations from 1871 to 1882.

# [ABLE D.--Mean Values of the Amplitude P, and of the First Maximum $\mu$ , reckoned from Local Apparent Midnight, for the First Three Orders of Harmonic Components of the Diurnal Inequality of Temperature at the Seven Observatories of the Meteorological Office, for each Month of the Year and for the entire Year, obtained from Twelve Years' Observations from 1871 to 1882.

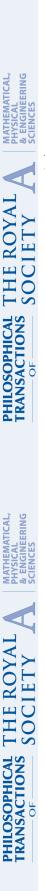
Кем.		288 119 601 501 500 501 501 501 501 501 501 501 5	.38
Арегдееп.		;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;	.27
Stonyhurst.		8 7 1 1 4 2 2 4 5 5 5 6 5 6 7 6 7 6 7 6 7 6 7 6 7 6 7 6	.32
Glasgow.	P.;.	011.00 01.00	08.
.hanomlaH		111 128 128 128 128 129 129 120 120 120	.56
Armagh.		2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	67.
Valencia.		11. 11. 11. 12. 12. 13. 13. 13. 13. 13. 13. 13. 13. 13. 13	.23
.wəД		.84 .95 .95 .93 .93 .33 .19 .19 .19 .140 .175	-84
.пөөрлөфА		6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	.62
Stonyhurst.		27. 100 86. 84. 722. 724. 76. 100 100 100 100 100 100 100 100 100 10	.72
Glasgow.	ρŢ	55. 56. 59. 58. 57. 57. 69. 10. 13.	29.
Falmouth		440 440 441 441 441 441 441 441 441 441	09.
Armagh.	-	.62 1.02 1.02 7.79 4.49 4.41 1.00 1.00 1.08 .86	-74
Valencia.		669 660 660 660 660 660 660 660 660 660	.61
Кем.	-	1.62 4.53 4.53 4.53 6.53 6.53 6.53 1.62 1.62 1.63	4.46
Арегдееп.		11.02.00.00.00.00.00.00.00.00.00.00.00.00.	19.61
Stonyhurst.		1.31 1.811 1.813 1.813 5.560 4.524 4.500 1.633 1	3.39
.wogsafD	P <sub>1</sub>	11.09 3.77 44.55 44.75 44.55 45.75 11.53 11.53	3.23
Falmouth.		0.10 0.10	2:35
.dgsmrA		11.2.4.2.4.4.4.8.2.1.1.0.2.2.4.2.2.4.2.2.4.2.2.4.2.2.2.4.2.2.2.2.1.2.1	3.24
Valencia.	-	0.91 1.34 2.27 3.02 3.03 3.01 3.01 1.24 0.97	2.58
Months.		January	Mean 12 years

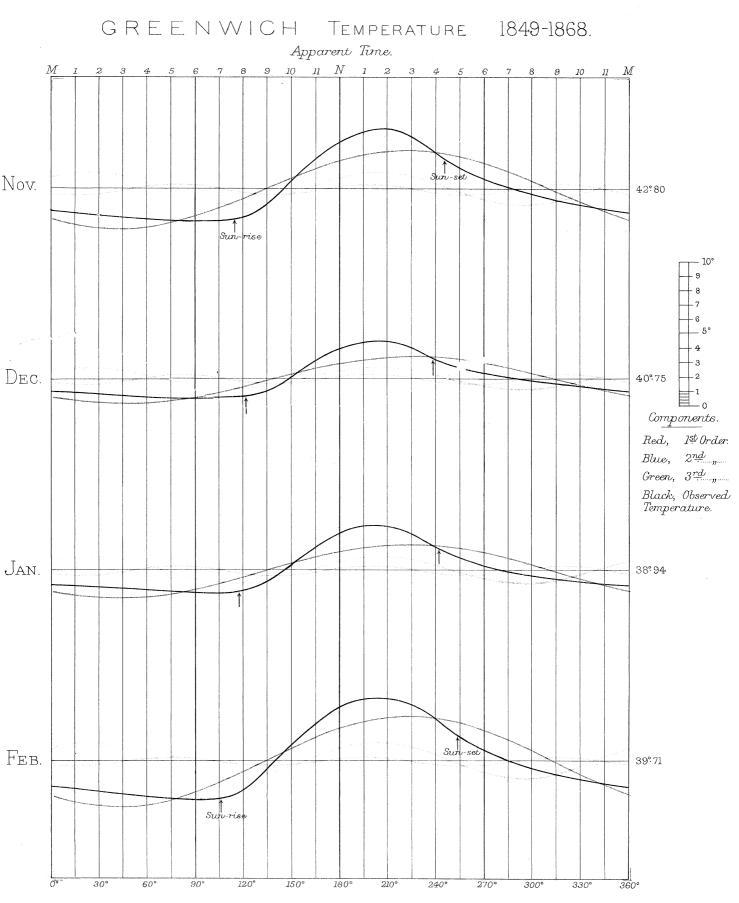
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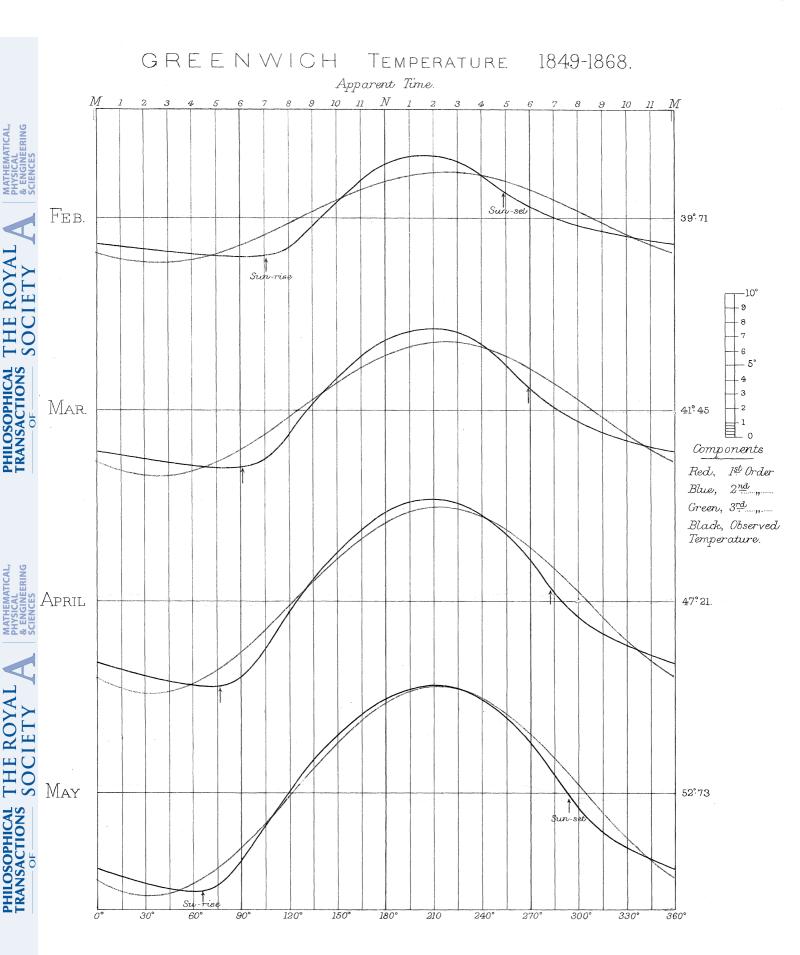
# HOURLY OBSERVATIONS OF AIR TEMPERATURE AND PRESSURE.

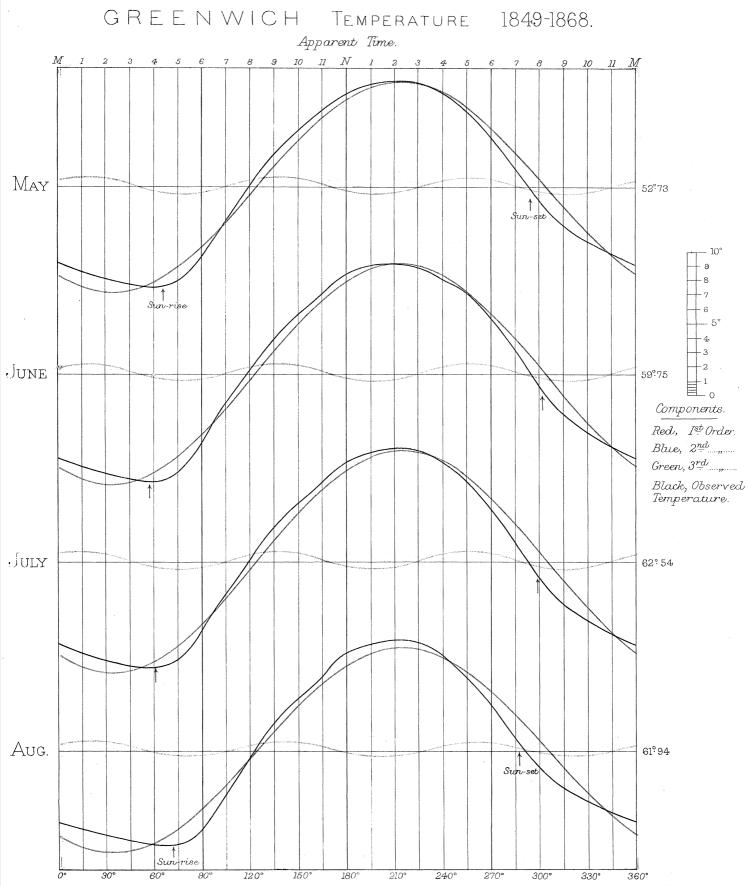
TABLE E.—Mean Monthly Values of the Equation of Time, the Sun's Declination and Right Ascension, and of the Apparent Time of Sun Rise, at Greenwich and at the Seven Observatories for each Month of the Year. The time is in Angular Measure.

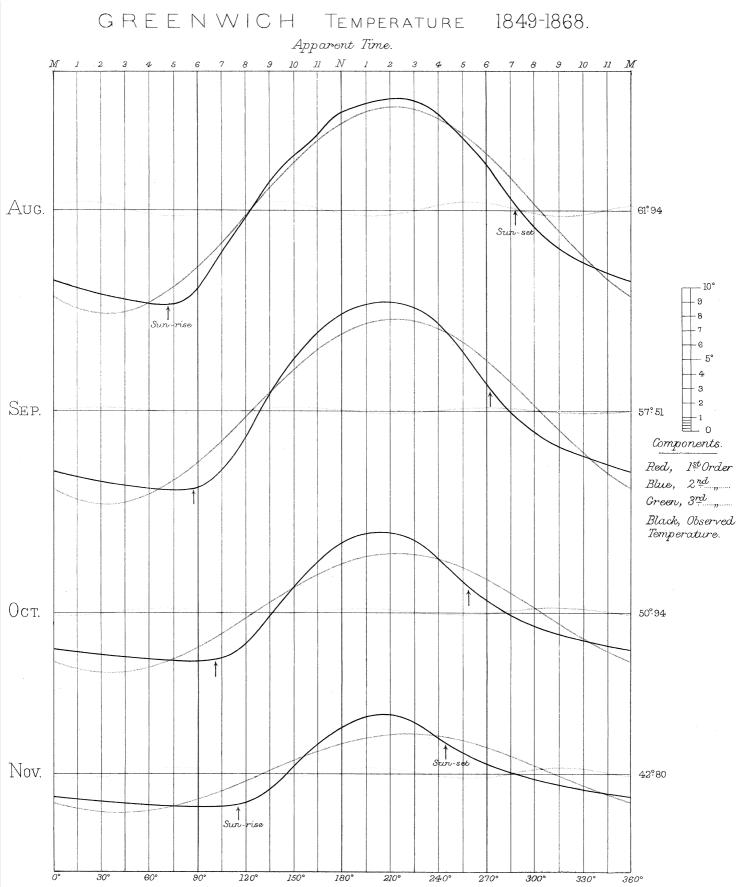
	57	17	16 30	35	37.	13 48	23 70	27	22 00
Dec.	S.23 265 265	122 1 15 5	122 1 15 5	125 3 13	120 £	$\begin{array}{c} 131 \\ 9 \end{array}$	128 E	126 2 12 3	122 5 15
Nov.	30.00	51	51	15 39	38	14 20	36 37	49	17 35
Ň	8.18 234	114	114 20	1117	113	121 14	119	117	115 19
Oct.	0.52	18	18 40	20 17	46 59	59 58	19 15	34 47	29
O	.80 80 80 80 80 80 80 80	101	101 29	102	100	103	103	102	101 29
Sept.	26 0	32 16	$\frac{32}{17}$	14 54	42 36	44 35	26 52	$\frac{10}{24}$	29 50
- S2	N. 22 173	86	86	86 38	86 42		85 36	86 38	86
August.	24° 0 48° 0 0 0 0	212	22	43 43	12	22 42	8	20	466
Aug	0 0 N.13 143	72	72	70	73	68	68	70	72
July.	20,	57	58 40	3	25	12	13	23 47	27
J.	 	60	60	52	62		70 70 70 10	57 56	60
June.	<b>'400</b>	31	$\frac{32}{41}$	11 18	120	30 59	52 16	$\begin{array}{c} 25 \\ 48 \end{array}$	56 14
Ju	N.23 N.23	57	57 61	75 40 60	5.00 63 63	4 で 8 で	50	70 70 60 80	56
May.	, 22 & O	28 9	$\begin{array}{c} 28 \\ 10 \end{array}$	13 47	37 29	29 28	145	42	4.8
M	+ 0 N.17 55	66 56	66 56	64 53	0,0	60 0 0	62	63 53	66 55
April.	′01 <del>1</del> 2 0	25	20	11 3	56 45	18 44	4 T	చ్చు. ప్రత్యే	8 63
Ap	N. 9 26	77	77	76 45	48	74 41	25 43 43	75	77
March.	<b>140</b>	20	02.23 83.33	8 vo	16	39 46	30 30 80	25 25 25	1 22
Ma	S. 1.2° 356	91	91 37	91	91 38	16	91	91 34	91 37
Feb.	31, 0 0	24 51	52	29	37	23	23	16	40
ř.	S.12 326	106	$\begin{vmatrix} 106 \\ 25 \end{vmatrix}$	104	105	110	$\begin{array}{c} 109 \\ 21 \end{array}$	$\begin{vmatrix} 108 \\ 22 \end{vmatrix}$	106
Jan.	0 38 25,	14 53	13 54	31	116 49 19 13	42	123 46 13 29	121 40 15 1	119 43 17 27
Ja	S. 20 38 296 0	118	118	121	116	125	123	121	119
		rise A. T. merid. alt.	. T. . alt.	. T. alt.	. T.	.T.	. T.	. T. . alt.	.T.
	· · ·	rise A	rise A. T. merid. alt.	rise A. T. merid. alt.	rise A. T. merid. alt.	rise A. T. merid. alt.	rise A. T. merid. alt.	rise A. T. merid, alt.	rise A. T. merid. alt.
	Apparent time = mean time Sun's declination	Sun rise A. T. , merid. alt.	1111	, ;; H H	, , H	; H H	; ; H H	" r	; ; H H
	mear			THE PROPERTY AND ADDRESS OF THE PARTY OF THE			CONTRACTOR OF THE PERSON OF TH	• •	
:	ne == tion de.	9' N.	3′ N. 3′ W.	1. N. S. W.	z's	У. W.	3, W.	1,' 9,' ₩.	& ĕ.ĕ. ⊗.ĕ.
	t tin clina ıgitu	ch. 11° 2% 0° (	1° 28 0° 19	rst. 33° 5. 2° 28	h. 50° 9′ 5° 4′	n. 7° 1( 2° 6′	55° 55° 55° 55°	.4.9 9.99	
	Apparent time = Sun's declination Sun's longitude.	Greenwich. Lat. 51° 29' N Long. 0° 0'	Kew. Lat. $51^{\circ}$ 28' N Long. $0^{\circ}$ 19' W	Stonyhurst. Lat. 53° 51′ N Long. 2° 28′ W	Falmouth. Lat. 50° 9′ N. Long. 5° 4′ W.	Aberdeen. Lat. 57° 10' N. Long. 2° 6' W.	Glasgow. Lat. 55° 53' N Long. 4° 18' W	Armagh. Lat. 54° 21′ N Long. 6° 39′ W	Valencia. Lat. 51° 55′ N Long. 10° 18′ W
	App Sun Sun	Gree Lau	Kew. Lat. Long	Stor Lai Loi	Falr La Lo	$_{ m Lab}$	Glas La Lo:	Arn La Lo	Vale La Lo

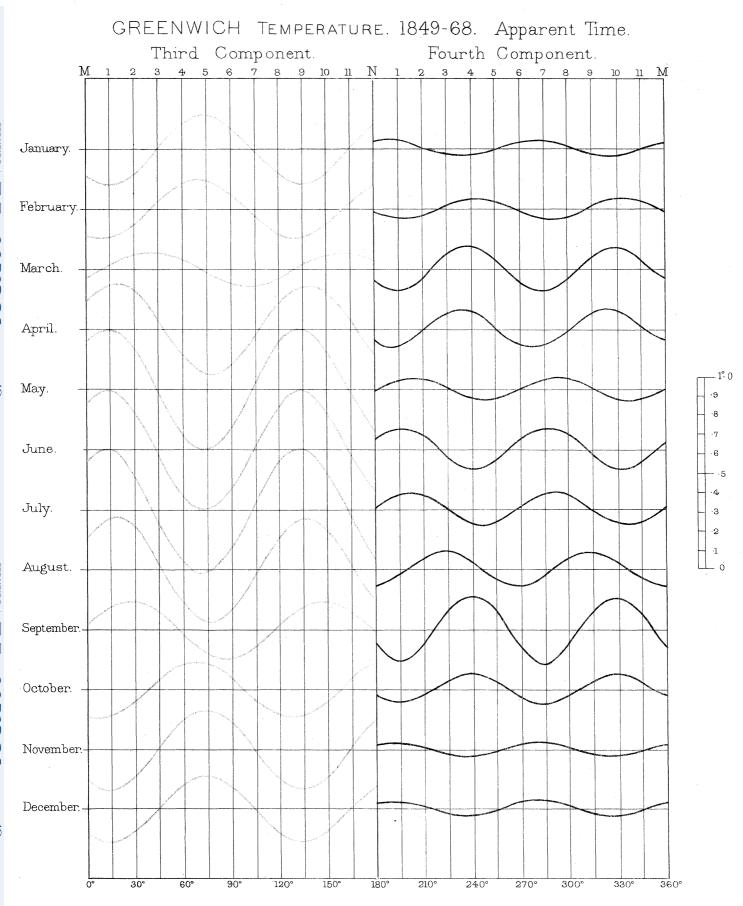












GREENWICH TEMPERATURE 1849-1868. Apparent Time. 8 9 10 11 N 1 2 3 4 5 6 7 8 9 10 11 M FEB. 39°.71 Sun-rise Components MAR. 41° 45 Red, 1st Order Blue, 2nd, Green, 3rd Black, Observed Temperature. 47°21. APRIL A MANDER OF THE SECTION OF THE SECTI 52:73 Su-rise 330° 360° 60° 270° 300° 150° 180° 210 240° 30° 90° 120°

GREENWICH TEMPERATURE 1849-1868. Apparent Time. 9 10 11 N 1 2 3 4 5 6 7 8 9 10 11 M MAY 52°73 Sun-rise Components. JUNE 59°75 Red, Ist Order. Blue, 2nd Green, 3rd Black, Observed Temperature. ULY 62°54 AUG. 61°94 Sun-set Sun rise 30° 90° 60° 120° 150° 180° 210° 240° 270° 300° 330° 360°

GREENWICH TEMPERATURE 1849-1868. Apparent Time. 9 10 11 M Aug. 61°94 Sur-rise SEP. 57°51 Components. Red, 1st Order Blue, 2nd Green, 3rd Black, Observed Temperature. 50°.94 Sur-set 42°80 Sun-rise 30° 60° 90° 120° 150° 300° 180° 210° 240° 270° 330° 360°